

# An efficient way to reduce losses of left-handed metamaterials

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We propose a simple and effective way to reduce the losses in left-handed metamaterials by manipulating the values of the effective parameters  $R$ ,  $L$ , and  $C$ . We investigate the role of losses of the short-wire pairs and the fishnet structures. Increasing the effective inductance to capacitance ratio,  $L/C$ , reduces the losses and the figure of merit can increase substantially, especially at THz frequencies and in the optical regime.

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## INTRODUCTION

The recent development of metamaterials with negative refractive index has confirmed that structures can be fabricated and interpreted as having both a negative effective permittivity,  $\epsilon$ , and a negative effective permeability,  $\mu$ , simultaneously. Since the original microwave experiments for the demonstration of negative index behavior in the split ring resonators (SRRs) and wires structures, new designs have been introduced, such as the short-wire pairs and the fishnet, that have pushed the existence of the negative refractive index into optical wavelengths [1, 2, 3]. However, both experiment and simulation results show that losses increase as the frequency increases. The transmission loss,  $1 - T$ , at low frequencies [4] is small (of the order of 1-5 dB/ $\lambda$ ), while as the frequency increases the loss increases, approaching values of almost 30 dB/ $\lambda$  at infrared frequencies [5]. Another factor to measure loss, namely the figure of merit, is the ratio of real and imaginary parts of the refractive index,  $|\text{Re}(n)/\text{Im}(n)|$ , drops from the order of 100 for SRRs at microwave frequency to 0.5 for the fishnet structure at optical frequency [6, 7]. So loss becomes a serious problem, which limits the potential applications of metamaterials such as perfect lens [8, 9]. Therefore we need to determine ways to reduce the losses, especially at high frequencies.

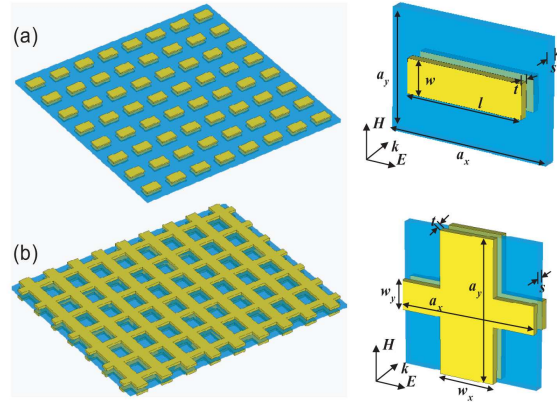


FIG. 1: Geometries for short-wire pair arrays (a) and the fishnet structure (b). Both consist of a patterned metallic double layer (yellow, usually Au) separated by a thin dielectric (blue).

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## NUMERICAL SIMULATIONS

In this manuscript, we tackle this problem and study the losses in the fishnet structures through numerical simulations. Our numerical simulations were completed with CST Microwave Studio (Computer Simulation Technology GmbH, Darmstadt, Germany), which uses a finite-integration technique. In Fig. 1, we show the structures of the short-wire pairs and the fishnet designs. The fishnet structure consists of double layer of infinite long metallic wire arrays along two orthogonal directions spaced by a dielectric spacer [10, 11, 12]. The wires along the magnetic field  $\mathbf{H}$  direction act as a magnetic resonator, providing negative permeability  $\mu$  due to anti-parallel currents induced by the magnetic field of the incident electromagnetic wave. The wires along the electric field  $\mathbf{E}$  direction of the incident electromagnetic wave excite the plasmonic response and produce negative permittivity  $\epsilon$  up to the plasma frequency. The fishnet structure has an intrinsic relation with the short-wire pairs [13, 14, 15] and the split ring resonator (SRRs) structure. The short-wire pairs geometry can be viewed as an extreme case of a two-gap SRR ring [16] shrunk along the direction of the gaps. By adding continuous wires, which provide negative permittivity  $\epsilon$ , one is able to obtain the negative refractive index,  $n$  [15]. The fishnet structure can be obtained by increasing the width of the short wires,  $w$ , to form continuous wires along the  $\mathbf{H}$  direction and by adding other continuous wires along the  $\mathbf{E}$  direction [11]. The transformation from the two-gap SRR to the short-wire pairs and the fishnet structure has been studied elsewhere [16]. Before we present the results for the fishnet structures, we will present the dependence of the

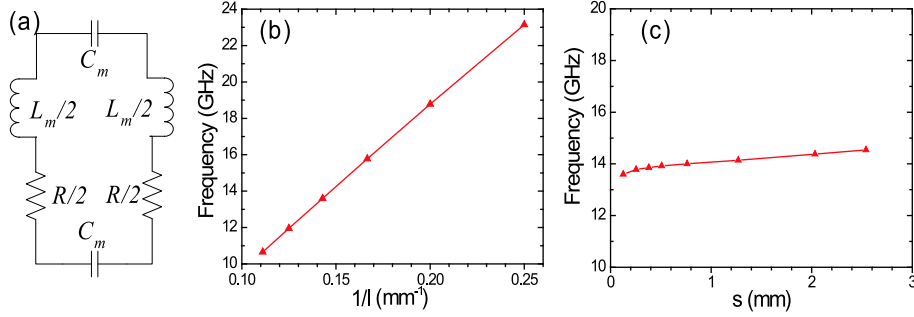


FIG. 2: (a) An effective  $RLC$  circuit of the short-wire pairs structure shown in Fig. 1(a). (b) Linear dependence of the magnetic resonance frequency,  $f_m$ , on the length of the short-wire  $l$ . The other parameters are given by  $s = 1.5$  mm,  $a_x = 9.5$  mm,  $a_y = 20$  mm,  $\epsilon_r = 2.53$ . (c) Dependence of the magnetic resonance frequency,  $f_m$ , on the thickness of the dielectric spacer,  $s$ . ( $w = 1$  mm,  $l = 7$  mm,  $a_x = 9.5$  mm,  $a_y = 20$  mm,  $\epsilon_r = 2.53$ ).

magnetic resonance frequency on the width and its length of the short-wire pair. The short-wire pairs give a magnetic resonance response [13, 17], and can give a negative  $\mu$ . The short-wire pairs structure has an interesting property that the resonance frequency,  $f_m$ , depends only on the length of the short-wire,  $l$ , among the geometric parameters,  $l, s, w$  and  $t$ . As shown in Fig. 2(b), the resonance frequency  $f_m$ , is a linear function of  $1/l$ , over a large range of  $l$ . On the other hand, we barely observe any change of  $f_m$  with the other parameters,  $s, w$  and  $t$ . Figure 2(c) shows the the magnetic resonance frequencies  $f_m$  as the separation  $s$  increases from 0.2 mm to 2.6 mm. One can see that although  $s$  increases by a factor of 13,  $f_m$  only changes around 5% (from 13.6 to 14.54 GHz). This is due to the fact that the capacitance,  $C_m = \epsilon_r(l \cdot w)/2s$ , and the inductance,  $L_m = \mu_r(l \cdot s)/w$ , have the opposite dependence on the width,  $w$ , and separation between the two short-wire pairs,  $s$  ( $s$  is also the thickness of the dielectric spacer) as shown in Fig. 1(a). The total capacitance of the series  $RLC$  circuit is given by  $C = \frac{1}{2}C_m$  and the total inductance is  $L = L_m$ , so the resonance frequency is given by  $\omega_m = 1/\sqrt{LC} = 2c_0/l\sqrt{\epsilon_r\mu_r}$ , where  $c_0$  is the speed of light in the vacuum and  $\epsilon_r$  and  $\mu_r$  are the dielectric function and relative permeability of the dielectric spacer, respectively [16]. As a transformation of the short-wire pairs structure, the fishnet structure has the same property, i.e., the resonance frequency  $f_m$  does not depend on the separation,  $s$ . This property gives us the opportunity to change  $L$  and  $C$  simultaneously without affecting the resonance frequency by changing the separation,  $s$ . The short-wire pairs structure does not give a negative  $n$ , but they give a negative  $\mu$ , all the way to the optical wavelengths [13, 17]. We will concentrate all of our efforts in trying to reduce the losses in the fishnet design, which is the best design so far, for giving a negative  $n$  at the optical frequencies [6, 18].

The magnetic element of the fishnet structure (i.e. the long wires along the  $\mathbf{H}$  direction), can be modeled as the same series  $RLC$  circuit as the short-wire pairs shown in Fig. 2(a). As we know, the loss of the  $RLC$  circuit depends upon the value of each circuit element and the quality factor,  $Q$ , is given by  $Q = \frac{1}{2R}\sqrt{\frac{L}{C}}$ . By increasing the inductance  $L$  or reducing the resistance  $R$  and capacitance  $C$ , we are able to increase the  $Q$ -factor and therefore reduce losses.

The resistance of the metallic structure depends on the conductivity and the frequency because of the skin effect, so we can choose a good conductor such as copper, silver or gold to reduce the resistance [19]. The inductance,  $L$ , and the capacitance  $C$ , strongly depend on the geometric parameters of the structure and are relatively easy to be changed by modifying these parameters.

In the fishnet structure, the long wire along the  $\mathbf{H}$  direction is like a magnetic resonator, which provides the negative  $\mu$  by introducing a magnetic resonance over a finite frequency band. The magnetic permeability is given by  $\mu = 1 - A\omega^2/(\omega^2 - \omega_m^2 + i\omega\Gamma_m)$ , where  $\omega_m$  is the magnetic resonance frequency and  $\Gamma_m$  is the damping factor, which is inversely proportional to the effective inductance,  $\Gamma_m \propto R/L$  [20]. So we are able to reduce the loss by increasing the inductance of the structure.

There is a special reason for choosing the fishnet structure in our study of the losses. Since the loss increases as the magnetic resonance frequency increases [2, 3, 5], we must compare the loss of metamaterials with different geometric parameters at the same resonance frequency. Because the fishnet structure has an intrinsic relation with the short-wire pairs, the magnetic resonance frequency  $f_m$  does not change with the thickness of the dielectric spacer,  $s$ . By increasing  $s$ , the effective inductance  $L$  will increase linearly with  $s$ , while the effective  $C$  decreases simultaneously with  $s$  and the product  $LC$  does not change, therefore, we are able to compare the losses at a fixed frequency. We

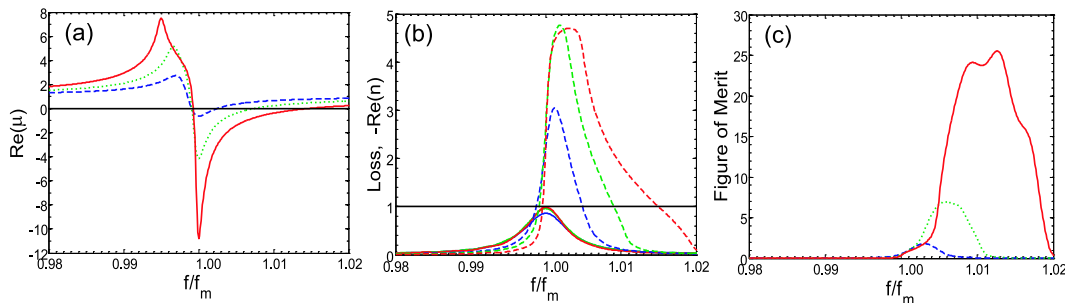


FIG. 3: (a) Effective permeability for the fishnet structure for three different widths of the dielectric spacer,  $s=0.25$  (blue dashed),  $0.5$  (green dotted) and  $1.0$  mm (red solid), respectively. The frequency  $f$  is normalized by the magnetic resonance frequency  $f_m$  ( $f_m = 9.701, 9.689$  and  $9.604$  GHz for  $s=0.25, 0.5$  and  $1.0$  mm, respectively). (b) The normalized loss,  $(1 - R - T)/(1 - R)$  (solid), and the real part of refractive index,  $-\text{Re}(n)$  (dashed), as a function of the normalized frequency. (c) Figure of Merit,  $|\text{Re}(n)/\text{Im}(n)|$ , versus  $f/f_m$ .

first studied the reduction of losses for the fishnet structure at microwave frequencies. In Fig. 3(a) we present the effective permeability  $\mu$  as a function of the normalized frequency. One can see that the magnitude of the magnetic resonance is significantly increased as the width of the dielectric spacer  $s$  increases. Fig. 3(b) shows the real part of the effective refractive index,  $-\text{Re}(n)$ , calculated from numerical simulations employing a retrieval procedure [21, 22], and the normalized losses, defined as the ratio of  $(1 - R - T)/(1 - R)$ , where  $T$  and  $R$  are the transmittance and reflectance, respectively. One can see that, as the separation  $s$  increases from  $0.25$  to  $1.0$  mm, the frequency with a given value of  $\text{Re}(n)=-1$ , shifts away from the center of the peak of the normalized loss. The underlying reason for this is the magnet resonance are becoming stronger as the inductance  $L$  increases, so that the frequency with a given value of  $\text{Re}(n)=-1$  moves away from the resonance peak where high loss occurs. Figure 3(c) shows that the figure of merit (FOM), which is defined as  $|\text{Re}(n)/\text{Im}(n)|$  and increases dramatically from 2 to 25. Further investigations show that an optimized fishnet structure at GHz frequencies can have a FOM in the order of 50. However, according to our simulations, the SRRs type metamaterial usually has a FOM larger than 100, due to the large effective inductance to effective capacitance ratio,  $L/C$ . It is very difficult to fabricate and characterize the SRRs at optical frequencies [2, 23], so we are forced to use the fishnet design, which gives  $n < 0$  for perpendicular propagation.

Another way to increase the inductance,  $L$ , is to increase the relative permeability,  $\mu_r$ , of the dielectric spacer. In order to keep the resonance frequency unchanged, we must decrease the dielectric constant,  $\epsilon_r$ , of the spacer accordingly, i.e., to keep  $\mu_r\epsilon_r$  as a constant. The magnitude of the magnetic resonance will increase dramatically as  $\mu_r$  increases. In order to avoid the periodicity effects [24, 25] due to the strong magnetic resonance, we limited the range of  $\mu_r$  to less than 2 and used a lossy material as the dielectric spacer with loss tangent,  $\tan\delta = 1.5 \times 10^{-3}$ . Figure 4(a) shows effective permeability,  $\mu$ , as the permeability of the spacer  $\mu_r$  changes from 1 to 2. One can see from Fig. 4(a) that the magnitude of the magnetic resonance increases by a factor of 2. In Fig. 4(b), the value of the real part of the refractive index,  $\text{Re}(n)$ , increases and moves away from the center of the loss peak. The figure of merit, shown in Fig. 4(c), also increases by a factor of 2 and has a maximum value of 6.

We also examined the losses of the fishnet structure in the infrared and the optical regimes. Similar to the microwave

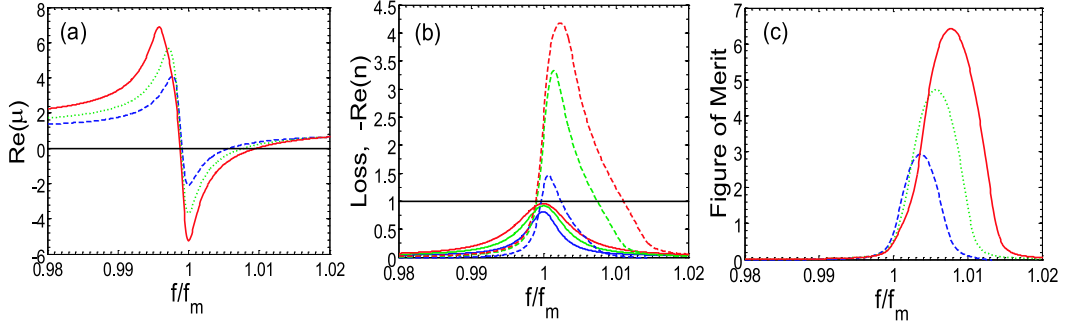


FIG. 4: (a) Effective permeability for the fishnet structure with permeability of the dielectric spacer  $\mu_r=1.0$  (blue dashed), 1.4 (green dotted) and 2.0 (red solid), respectively. The dielectric constant is  $\epsilon_r = 20/\mu_r$ . The frequency  $f$  is normalized by the resonance frequency  $f_m$  ( $f_m=2.323, 2.325$  and  $2.330$  GHz for  $\mu_r=1.0, 1.4$  and  $2.0$ , respectively). (b) The normalized loss and the real part of refractive index (dashed). (c) Figure of Merit.

frequency, the magnetic resonance become stronger as the separation,  $s$ , increases from 30 to 90 nm as shown in Fig. 5(a). Figure 5(b) shows the effective refractive index  $\text{Re}(n)$  versus the frequency at THz region. Notice that  $n < 0$  at 370 THz, which is in the optical regime. In Fig. 5(c), one can see that the figure of merit increases from 4.2 to 10.0 (peak value) as the separation,  $s$ , changes from 30 to 90 nm. At frequencies above 100 THz, the losses of the fishnet structure increase rapidly as the resonance frequency increases [6, 10, 26]. In our simulations, we manage to achieve a  $\text{FOM}=2.5$  at 620 THz ( $\lambda=484$  nm) using silver (The permittivity of silver is described by the Drude model with the plasma frequency,  $f_p=2181$  THz, and the damping frequency,  $f_c=14.4$  THz, which is 3.3 times the damping frequency of the bulk material to take into account the high loss in the thin layer of silver.) As a comparison, the best results so far in the optical regime, is  $\text{FOM}=0.5$  at  $\lambda=784$  nm [6]. So, our new fishnet design has reduced the losses and increased substantially the figure of merit.

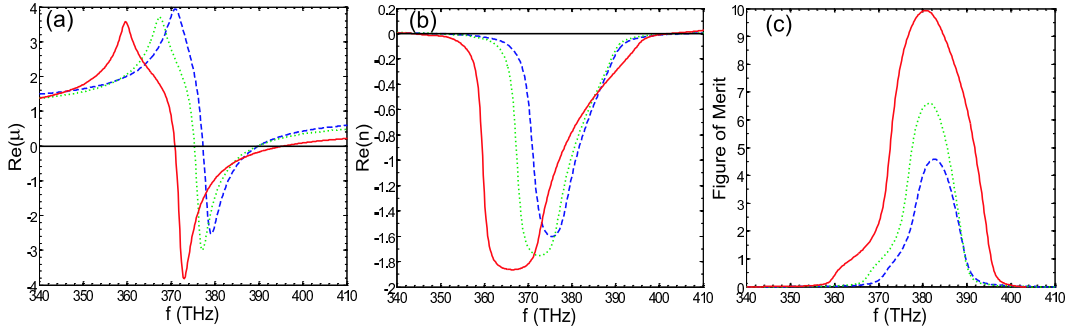


FIG. 5: (a) Effective permeability for the fishnet structure for three different widths of the dielectric spacers,  $s=30$  (blue dashed), 60 (green dotted), and 90 nm (red solid), respectively. The other parameters are given by  $w_x = 100$  nm,  $w_y=200$  nm,  $a_x=a_y=300$  nm,  $t=40$  nm,  $\epsilon_r=1.90$ . (b) The real part of refractive index (dashed). (c) Figure of Merit.

It is worth to point out that one can not increase  $s$  in the fishnet design arbitrarily. The maximum value of  $s$  is limited by two facts. First, it is restricted by the unit cell size,  $a_z$ , in the propagating direction. The unit cell size,  $a_z$ , is limited by the homogenous assumption of left-handed materials, i.e.  $a_z \ll \lambda$ , and also by the requirement of negative permittivity, which is provided by the long wires along the electric field direction and will be diluted by a large unit cell. Second, according to our simulations, as  $s$  increases up to a certain value larger than the width of wires,  $w$ , the magnetic resonance will disappear. This is due to the fact that the short wires are decoupled from each other as  $s \gg w$ .

## CONCLUSIONS

In summary, we proposed a simple and efficient way to reduce losses in the left-handed metamaterial designs by increasing the inductance to the capacitance ratio,  $L/C$ . We found that the figure of merit of the fishnet structure can be as large as 50 at microwave frequencies. Our method is also valid in the infrared and in the optical regime, we

should be able to obtain a figure of merit of 2.5 at  $\lambda=484$  nm, which improved the figure of merit by a factor of 5, comparing with the best result at  $\lambda=784$  nm so far. Although our approach is presented using the fishnet structure, it's a generic method and can also apply to other type left-handed material designs such as SRRs.

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